

Specialist Mathematics Units 3,4
Test 1 2018

Calculator Free
Functions

STUDENT'S NAME _____

DATE: Monday 26 February

TIME: 50 minutes

MARKS: 56

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser, formula sheets

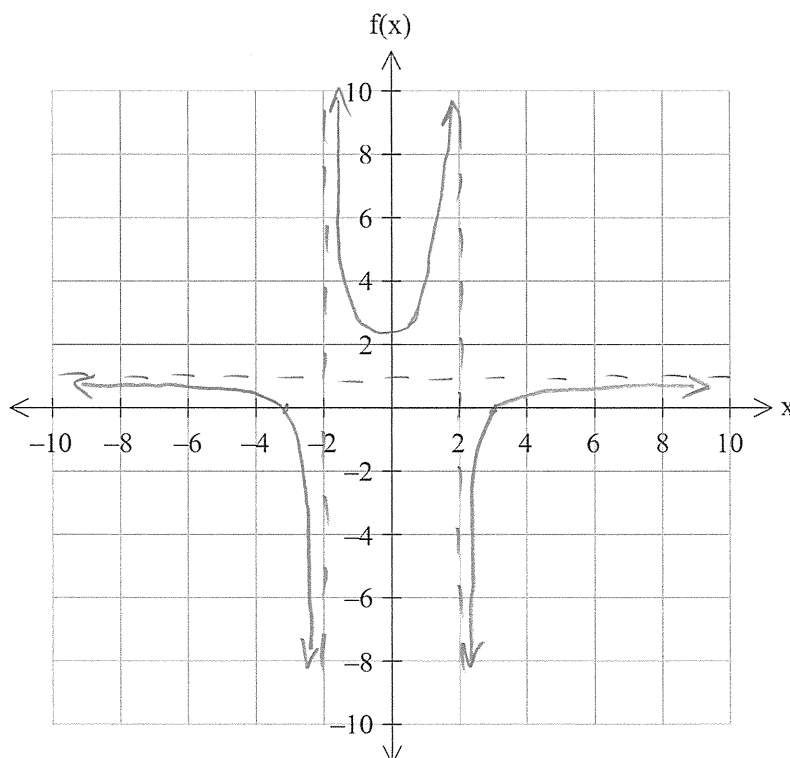
Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

1. (5 marks)

Sketch $f(x) = \frac{x^2 - 9}{x^2 - 4}$ on the axes below.

$$= 1 - \frac{5}{x^2 - 4}$$

$$\begin{array}{r} 1 \\ x^2 - 4 \overline{) x^2 - 9} \\ \underline{-(x^2 - 4)} \\ -5 \end{array}$$



2. (12 marks)

Solve each of the following equations.

(a) $|x+1|=2|x-2|$ [4]

$$(x+1)^2 = 4(x-2)^2$$

$$x^2 + 2x + 1 = 4x^2 - 16x + 16$$

$$0 = 3x^2 - 18x + 15$$

$$0 = x^2 - 6x + 5$$

$$0 = (x-5)(x-1)$$

$$x = 1, 5$$

(b) $|3x-7|+2x=5$ [4]

$$|3x-7| = 5-2x$$

$$3x-7 = 5-2x \qquad -(3x-7) = 5-2x$$

$$5x = 12 \qquad 2 = x$$

$$x = \frac{12}{5}$$

(c) $\left| \frac{x+k}{x-k} \right| \geq 3$ where k is a positive constant [4]

$$\frac{x+k}{x-k} = 3 \qquad \frac{x+k}{x-k} = -3$$

$$x+k = 3x-3k \qquad x+k = -3x+3k$$

$$4k = 2x \qquad 4x = 2k$$

$$2k = x \qquad x = \frac{k}{2}$$

TEST $x=0$

$$\left| \frac{k}{-k} \right| \geq 3$$

$$|-1| \geq 3$$

$$1 \geq 3$$

FALSE



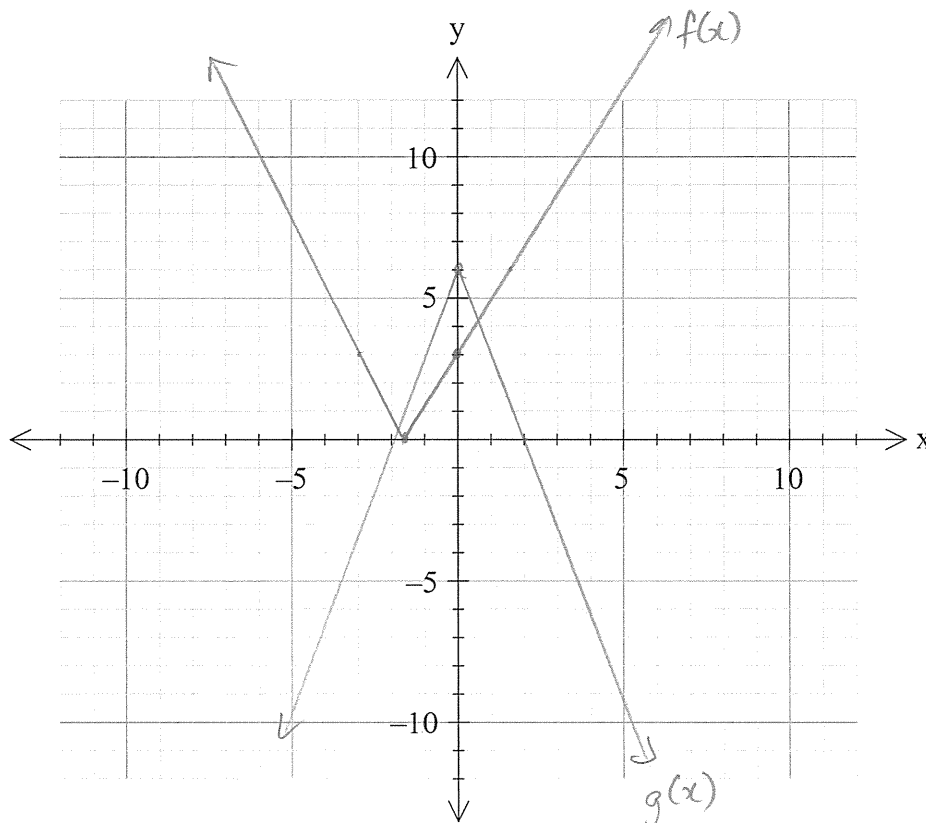
$$\frac{k}{2} \leq x \leq 2k \quad x \neq k$$

3. (8 marks)

Given the functions $f(x) = |2x+3|$ and $g(x) = 6 - |3x|$

(a) sketch both functions on the same set of axes below.

[4]



(b) Hence, or otherwise, solve $|2x+3| + |3x| < 6$

[4]

$$|2x+3| < 6 - |3x|$$

$$2x+3 = 6-3x$$

$$5x = 3$$

$$x = \frac{3}{5}$$

$$-(2x+3) = 6+3x$$

$$-5x = 9$$

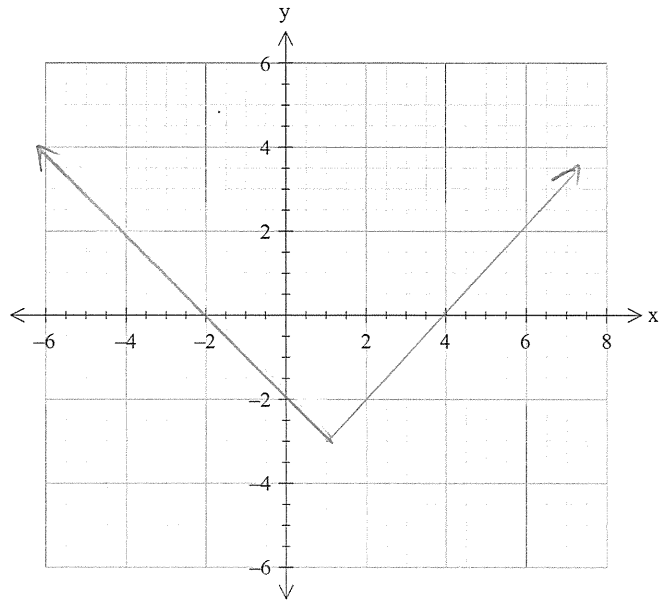
$$x = -\frac{9}{5}$$

$$-\frac{9}{5} < x < \frac{3}{5}$$

4. (7 marks)

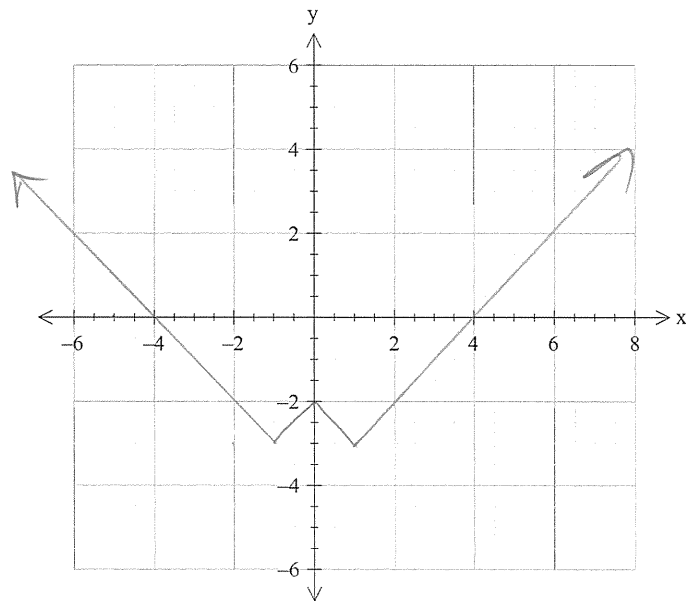
Given $y = f(x) = |x-1| - 3$

(a) sketch $y = f(x)$



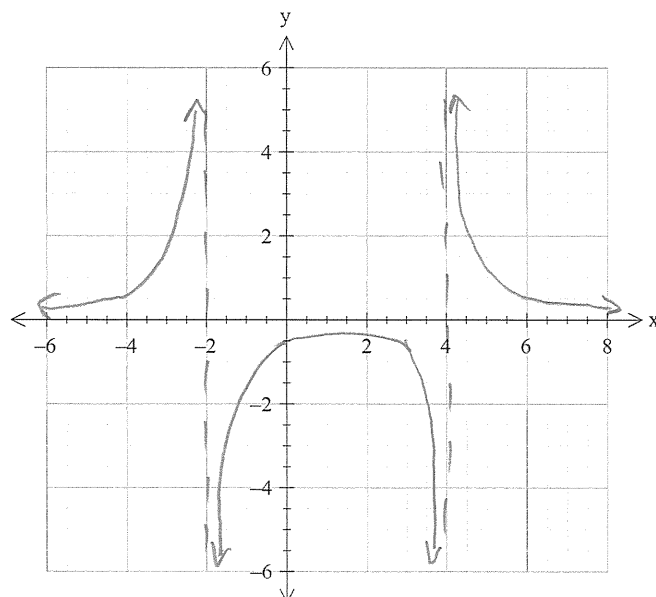
[2]

(b) sketch $y = f(|x|)$



[2]

(c) sketch $y = \frac{1}{f(x)}$



[3]

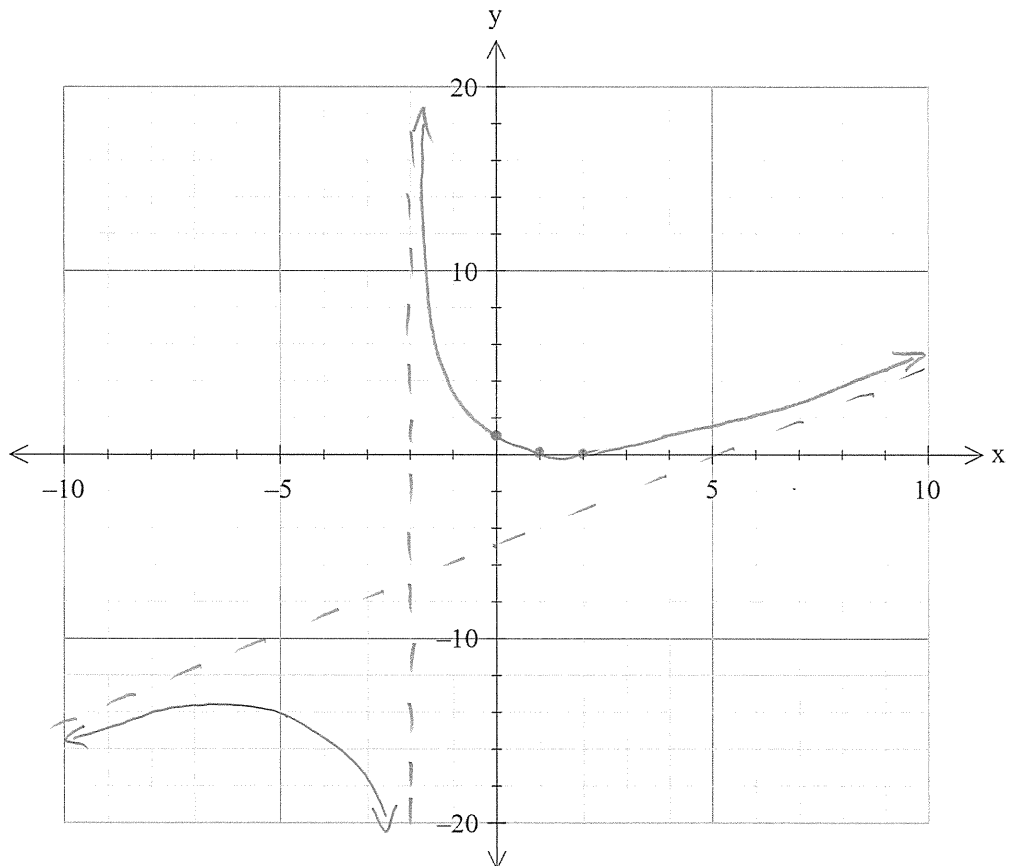
5. (5 marks)

Sketch $y = \frac{(x-1)(x-2)}{x+2}$ on the axes below.

(Note – you are not required to determine the exact coordinates of any stationary points)

$$\begin{array}{r} x-5 \\ x+2 \overline{) x^2 - 3x + 2} \\ \underline{-(x^2 + 2x)} \\ -5x + 2 \\ \underline{(-5x - 10)} \\ 12 \end{array}$$

$$y = x - 5 + \frac{12}{x+2}$$



6. (5 marks)

For the two functions $f(x) = \sqrt{2x+8}$ and $g(x) = x^2 + 3$

(a) determine the domain of $f(x)$

$$2x+8 \geq 0$$

$$x \geq -4$$

$$D: \{x: x \geq -4, x \in \mathbb{R}\}$$

[1] 2

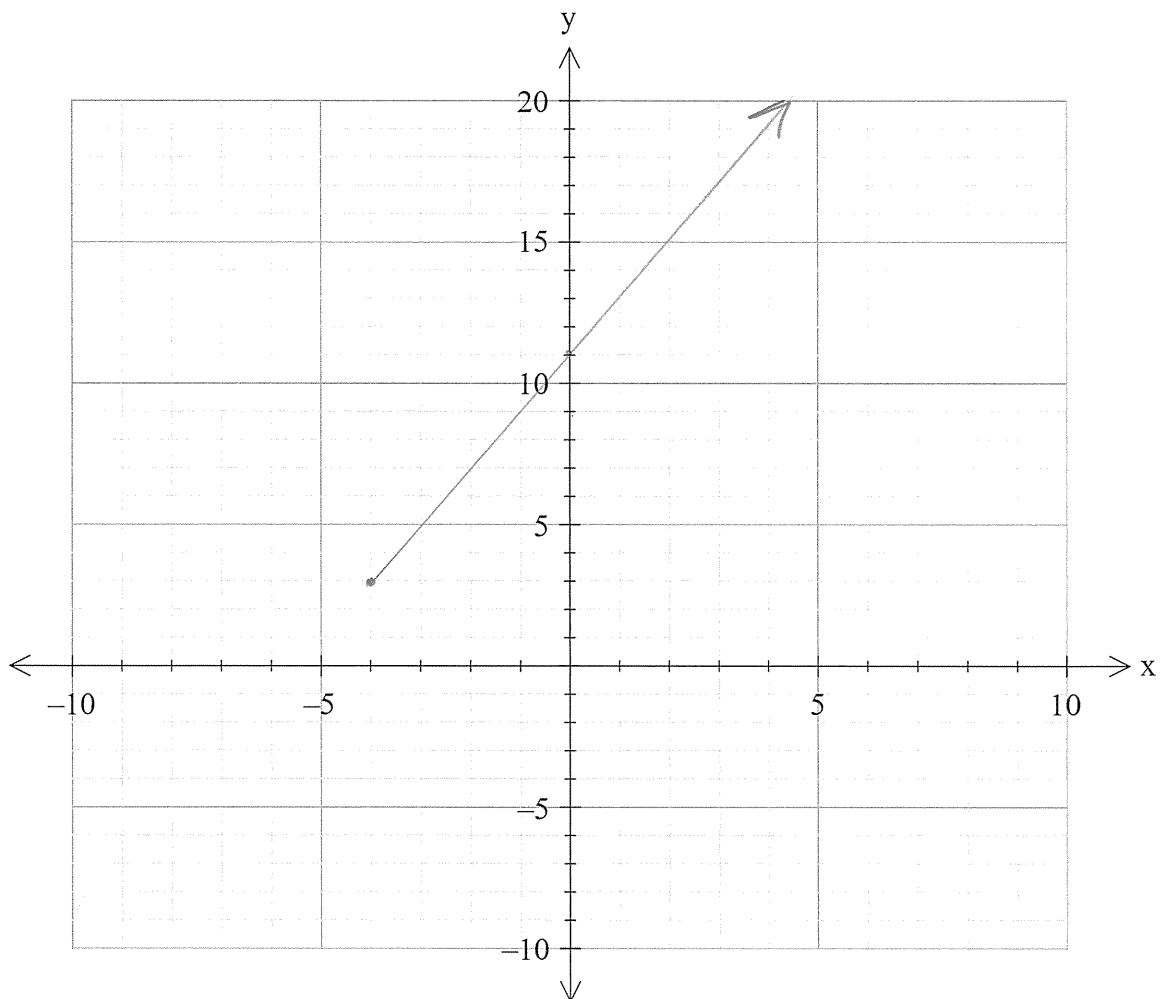
(b) determine the domain of $g \circ f(x)$

$$D: \{x: x \geq -4, x \in \mathbb{R}\}$$

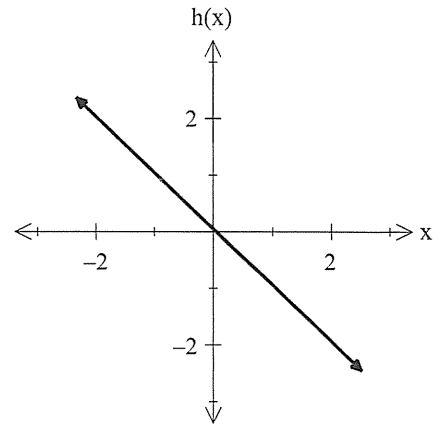
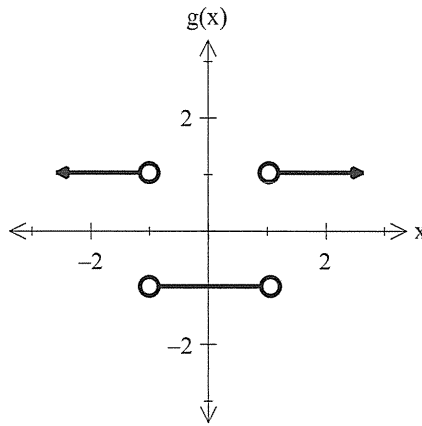
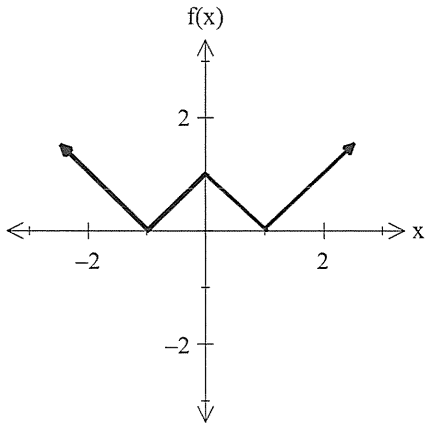
[2] 1

(c) sketch $y = g \circ f(x)$

[2]



7. ⁵
(6 marks)



(a) Using the graphs shown above, determine

(i) $f \circ f(-1)$ 1 [1]

(ii) $g \circ h(-1)$ DOES NOT EXIST [1]

(iii) $h^{-1} \circ f(0)$ -1 ~~[2]~~ 1

(b) Determine the range of $f \circ g(x)$ $R: \{y: y=0\}$ [2]

RANGE $g(x) = \pm 1$

DOMAIN $f \circ g(x) = \pm 1$

8. (8 marks)

For the functions $g(x) = \frac{1}{x}$ and $h(x) = \frac{x+1}{x-1}$

(a) determine $h^{-1}(x)$ in terms of $h(x)$

[3]

$$\begin{aligned}h &: y = \frac{x+1}{x-1} \\h^{-1} &: x = \frac{y+1}{y-1} \\xy - x &= y + 1 \\xy - y &= x + 1 \\y(x-1) &= x + 1 \\&= \frac{x+1}{x-1} \\h(x) &= h^{-1}(x)\end{aligned}$$

(b) show $g \circ h(x) = h(-x)$

[3]

$$\begin{aligned}g \circ h(x) &= \frac{1}{\frac{x+1}{x-1}} \\&= \frac{x-1}{x+1}\end{aligned}$$

$$\begin{aligned}h(-x) &= \frac{-x+1}{-x-1} \\&= \frac{-x+1}{-(x+1)} \\&= \frac{x-1}{x+1}\end{aligned}$$

(c) determine $h \circ h(x) = h \circ h^{-1}(x)$

[2]

$$= x$$